

# IDXX & MAR Report

Group 30



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**30 September 2025**

## Assessment Submission Form

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### Declaration of Authorship

I declare that all material in this assessment is my own work except where there is clear acknowledgement and appropriate reference to the work of others.

**Signed: Alexander, Karthik, Colm, Rory, Vishnu, Kenny**

**Date:** 30/09/2025

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## **Project Work Breakdown Structure by Member**

Below is a formal work breakdown structure, organized by group member, highlighting individual responsibilities, collaborations, and contributions to both analysis and presentation:

### **Alex**

- Solved Questions 1 to 6
- Led presentation and report writing for Questions 1 to 6
- Collaborated with Rory on Questions 2, 5, and 6 (analysis, presentation, and report)
- Participated in overall presentation creation and editing

### **Rory**

- Solved Question 6
- Collaborated with Alex on Questions 2, 5, and 6 (analysis, presentation, and report)
- Supported presentation and report writing for these questions
- Participated in overall presentation creation and editing

### **Colm**

- Solved Question 7 (analysis, presentation, and report)
- Prepared presentation and report for Question 7
- Authored the introduction and conclusion sections of the overall project report
- Participated in overall presentation creation and editing

### **Vishnu**

- Solved Question 8 (analysis, presentation, and report)
- Collaborated with Karthik to perform OLS regression and calculate stock betas for Q8
- Prepared presentation and report for Question 8
- Participated in overall presentation creation and editing

## **Karthik**

- Collaborated with Vishnu to perform OLS regression and calculate stock betas for Q8
- Solved Questions 9 and 10 (analysis, presentation, and report)
- Prepared presentation and report for Questions 9 and 10
- Provided conclusions for Questions 9 and 10
- Participated in overall presentation creation and editing

## **Kenny**

- Supported report writing for Questions 1 and 6 (analysis, presentation, and report)
- Prepared presentation for Questions 3 and 4
- Participated in overall presentation creation and editing

## Introduction

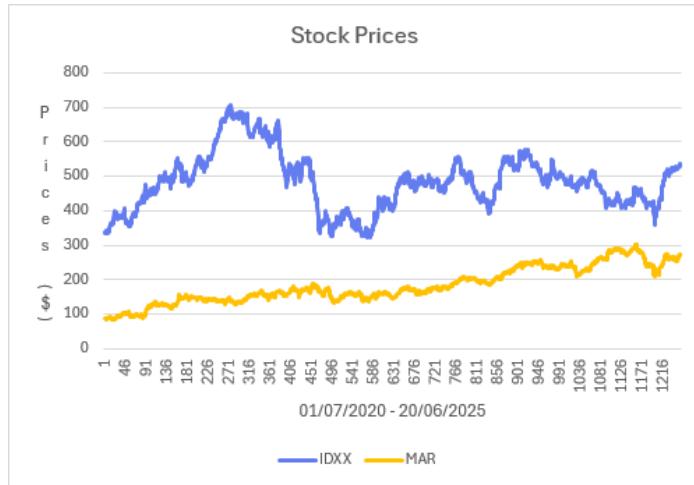
Balancing risk and return is the primary challenge for portfolio managers and retail investors in modern investment practice. This report will evaluate how an optimal portfolio can be constructed using two risky assets (IDXX and MAR) and a risk-free asset to maximise returns while applying theoretical concepts such as the ones formalised by (Markowitz (1952)). The two risky assets used for this report operate in different industries and this report will calculate the expected returns, variance, covariances and correlations between the two to identify a portfolio which will maximise the Sharpe ratio. IDXX and MAR are successful companies but operate with unique idiosyncratic risks. Thus, it is important to analyse them side by side as this allows us to examine the effectiveness of diversification and the relationship of the two stocks. By constructing the Capital Allocation Line (hereby CAL) and combining it with the risk-free asset, this report will highlight how every investor has access to the same set of opportunities with these three assets but will weight their portfolio differently based on their attitude to risk.

## **Q1 Company Information and Stock Price**

### **Commentary**

IDEXX (2025a) Laboratories is a multinational healthcare company that specialises in veterinary diagnostics and software for clinics, develops testing kits that ensure the safety of milk and drinking water. IDEXX (2025b) operates in 22 countries across six continents. It is listed on NASDAQ with a market capitalisation of around \$50.45 billion (Yahoo Finance, 2025). Their industry has been experiencing steady growth, with the global veterinary diagnostics market projected to grow at a Compound Annual Growth Rate (CAGR) of 10% from 2025 to 2030, currently valued at \$11.8 billion and expected to reach \$19 billion by 2030 (Grand View, 2025). As of Q2 Fiscal Year 2025, Diagnostics continues to drive growth for IDXX (2025c) Laboratories, generating a 10% year-on-year revenue gain.

Marriott International is a multinational corporation in the hotel industry that offers three key services for hoteliers: operating a franchise system, licensing their brand name, and charging a hotel management fee. Marriott operates and targets a broad range of market segments including luxury, premium, select, longer-stay, and collections, across six continents (Marriott International, 2025a). It is listed on NASDAQ and has a market capitalisation of \$70 billion (Yahoo Finance, 2025). 7% CAGR is anticipated for the hotel industry for the years 2025-2030 with the market valued at \$455.15 billion and expected to reach \$638.39 billion by the end of the period (Statista, 2025). The growth is driven by customers seeking a more premium service (Statista, 2025), aligning perfectly with Marriott International (2025b), considered an “innovative leader in luxury hospitality”.



Appendix A: Stock Prices

Appendix A demonstrates a significant difference in the volatility of stock prices over the five-year period between July 1, 2020, and June 30, 2025. IDXX's stock price peaked at \$705.76 on August 5, 2021, and reached its low at \$381.12 on October 14, 2022. Meanwhile, MAR reached a low of \$83.52 on August 3rd, 2020 and reached \$304.45 by February 10, 2025. While MAR shows steady returns and IDXX exhibits more volatile returns, MAR originates from a cyclical industry (Bodie et al., 2021), which is sensitive to consumer demand for hotels, and IDXX come from a defensive industry, showing strong performance during the COVID pandemic.

## **Q2 Daily Returns and Correlation**

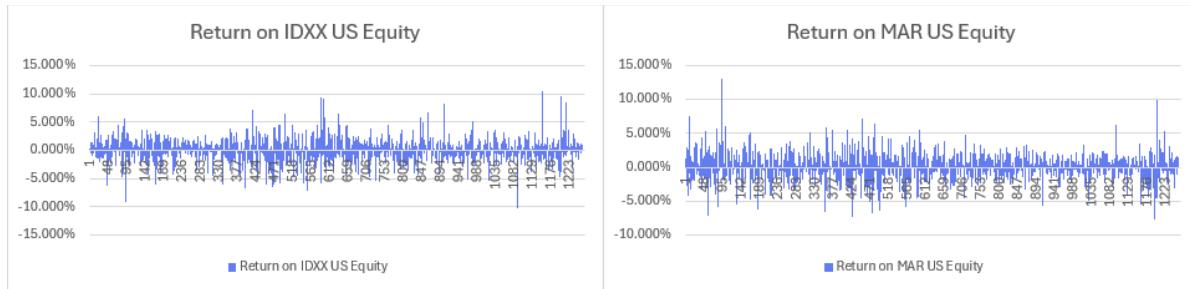
Daily data was imported to Excel from the Bloomberg terminal. Using these stock prices, we calculated the daily return on each stock using log returns where return  $r = \ln\left(\frac{P_n}{P_{n-1}}\right)$ .

Taking the mean of all values between the studied time period, we found the average daily return for IDXX and MAR. These were 0.037611% for IDXX and 0.091485% for MAR.

We can see that MAR had a more successful period of growth during this time period. MAR's daily return is just under two and half times that of IDXX. It is important to note that although the main crash due to the COVID-19 pandemic didn't occur during our given time range, MAR had been trading at ~\$140 per share before it crashed, meaning our opening price of \$86.62 on 1 July 2020 had room for rebound.

The volatility of each stock was calculated using Excel. These results came to be 0.02078789 for IDXX and 0.019645778 for MAR.

The basic principles of risk and return say that higher risk means higher return. However, MAR has had a higher return with a lower standard deviation throughout the time period.



### Appendix B: Daily Return Changes

Finally, we calculated the correlation between our two stocks and divided the covariance between the product of our two standard deviations. The correlation between IDXX and MAR was 0.299442597. This is a positive but mild correlation. Its R-squared of a little under 9% is quite low.

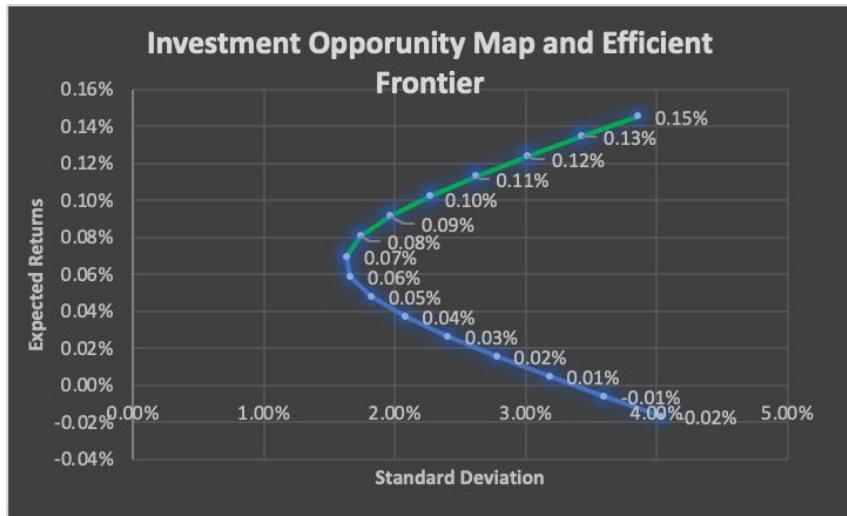
## **Q3/4 The Efficient Frontier & Investment Portfolio**

Using the daily log returns of both stocks, we calculated the mean return, standard deviation, and covariance between them. These statistics enabled us to calculate the portfolio returns and risks for various weight combinations of the two stocks, including cases with short sales and leverage.

We created a two-asset investment opportunity set, comprising all portfolios with different possible combinations of risk and return through various weights (Bodie et al., 2021). Notably, this examined the full risk-return profile of these assets, excluding the risk-free rate, and focuses on the relationship between IDXX and MAR. To create this opportunity set, we first found the expected average weighted returns of portfolios through the formula  $E[r_p] = w_B E[r_B] + w_S E[r_S]$

Next, we measured the risk of the portfolios, which accurately assesses the covariance of two assets within a portfolio as:

$$\Sigma_p = \sqrt{(w_B \sigma_B)^2 + (w_S \sigma_S)^2 + 2(w_B \sigma_B)(w_S \sigma_S) \rho_{BS}}$$



### Appendix C: Opportunity and Frontier

The investment opportunity set was mapped using the expected returns as the dependent variable and standard deviation as the independent variable. This revealed an efficient portfolio set marked in green, highlighting the most optimal portfolios which maximise returns for the same amount of standard deviation as those below them.

We located the data points on the frontier and found their corresponding weights. It revealed a stark observation that, without leverage or shorting, the best combinations to long hold would be 40% IDXX and 60% MAR, 20% IDXX and 80% MAR, and 0% IDXX and 100% MAR. Naturally, the weighting moves towards MAR. This explains why it is highly encouraged to leverage MAR at 120%, 140%, 160%, 180%, and 200% while shorting IDXX at a corresponding -20%, -40%, -60%, -80%, and -100%. It's assumed that the MAR stock price will generate consistent and strong returns, while IDXX is expected to underperform. Nevertheless, the portfolio that maximises an optimum amount of returns for the least amount of risk is, 40% IDXX and 60% MAR, and the portfolio that maximises an optimum return for the most amount of risk is, -100 IDXX and 200% MAR.

IDXX US Equity Weighted	MAR US Equity Weighted	Return	St Dev
200.00%	-100.00%	-0.02%	4.03%
180.00%	-80.00%	-0.01%	3.60%
160.00%	-60.00%	0.01%	3.18%
140.00%	-40.00%	0.02%	2.78%
120.00%	-20.00%	0.03%	2.41%
100.00%	0.00%	0.04%	2.08%
80.00%	20.00%	0.05%	1.82%
60.00%	40.00%	0.06%	1.66%
40.00%	60.00%	0.07%	1.63%
20.00%	80.00%	0.08%	1.74%
0.00%	100.00%	0.09%	1.96%
-20.00%	120.00%	0.10%	2.27%
-40.00%	140.00%	0.11%	2.62%
-60.00%	160.00%	0.12%	3.01%
-80.00%	180.00%	0.13%	3.43%
-100.00%	200.00%	0.15%	3.86%

## Appendix D: Optimal Portfolios

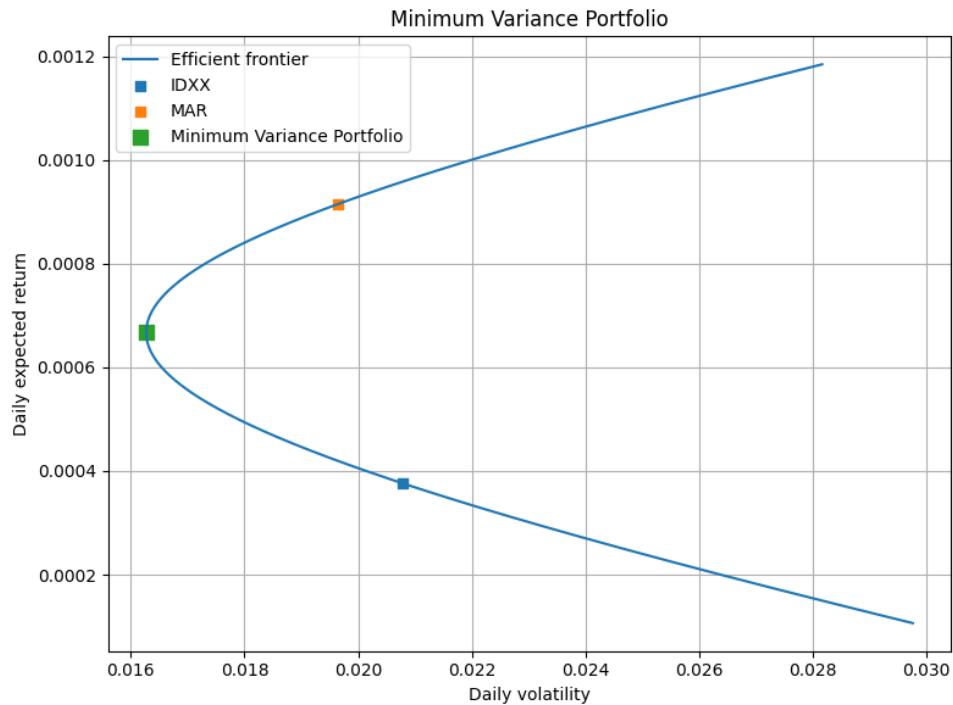
### Q5 The Minimum Variance Portfolio

We examined our efficient frontier graph from Q3. We found the minimum variance formula in two ways. Looking at the graph, the left-most point on the efficient frontier was our minimum variance portfolio. Next, we calculated it using a Python script. The minimum

variance portfolio only depends on risk, not returns. We used the formula  $W_I = \frac{\sigma_M^2 - \sigma_{IM}}{\sigma_I^2 + \sigma_M^2 - 2\sigma_{IM}}$

to calculate the weight of IDXX in our minimum variance portfolio. Our exact minimum variance portfolio weights were, IDXX 45.97% and MAR 54.03%. The return on this portfolio is 0.066717%, having a standard deviation of 0.016269 and a Sharpe Ratio of 0.0338972.

Looking at our minimum variance portfolio, diversification benefits are obvious. The overall risk of the MVP is at 1.6269%, lower than the individual risks of the IDXX at 2.079% and MAR at 1.965%. We have not maximised return or the Sharpe ratio; however, we have found the point on our efficient frontier with the lowest volatility.



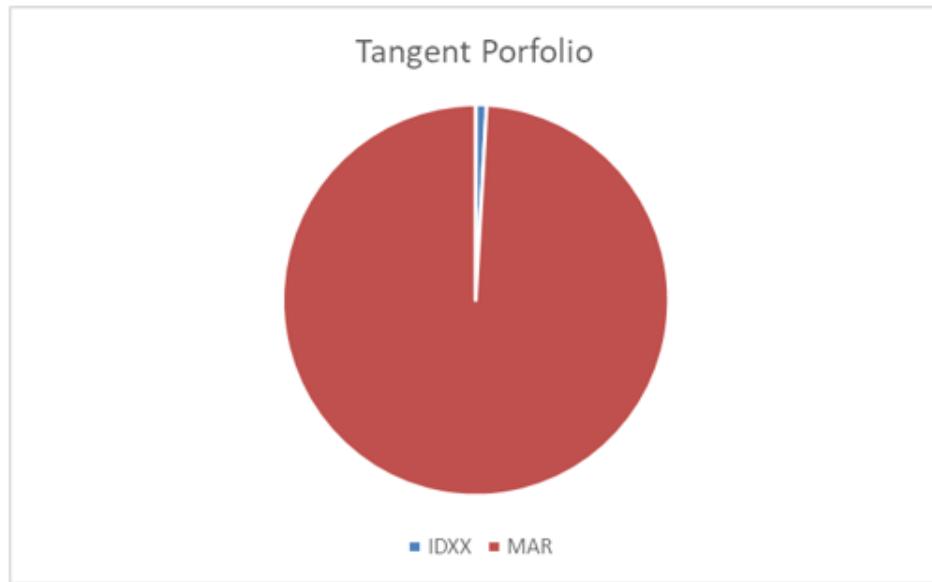
#### Appendix E: Minimum Variance Portfolio

## Q6 The CAL and Tangency Portfolio

Following this, we downloaded the daily risk-free rates from 01 July 2020 to 30 June 2025. We used the ‘Market Yield on U.S. Treasury Securities at 3-Month Constant Maturity (DGS3MO)’ (FRED, 2025) to get our risk-free rate for the period. We then calculated our daily mean rate.

In question 2, we had calculated IDXX and MAR mean daily return, standard deviation as well as their correlation. Using the DGS3MO we found that the daily risk-free rate was 0.115682%. The calculations were done by first building a variance and covariance  $2 \times 2$  matrix using our Q2 values. We denote this matrix as  $\Sigma$ . We then created a second  $2 \times 1$  matrix made up of the mean daily returns of our stocks minus the risk-free rate. Multiplying  $\Sigma^{-1}$  by this second matrix gave us our values, which we then normalised to equal 1. We then calculated our mean return, standard deviation, and Sharpe Ratio (Appendix).

Our final weights are 0.9114688% in IDXX and 99.0885312% in MAR.



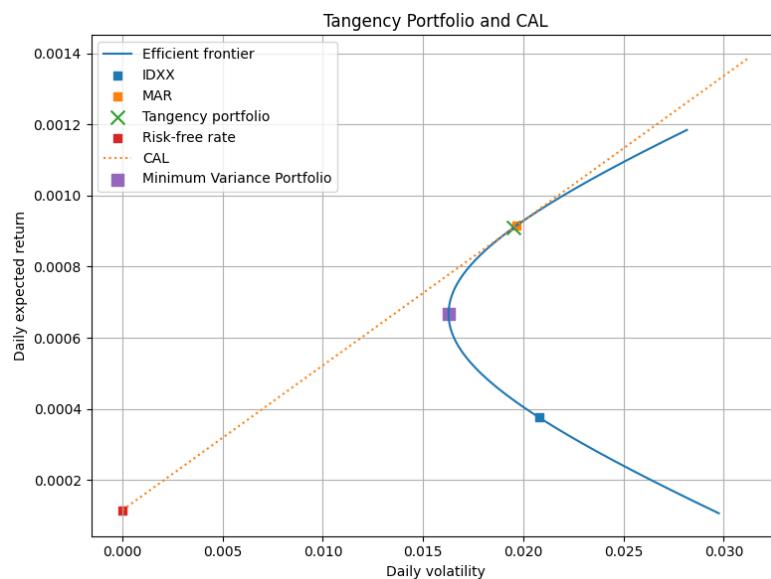
#### Appendix F: Tangent Portfolio Distribution

MAR dominates this portfolio due to its higher daily mean return and lower volatility.

We also used a Python script to graph and confirm our values. (Appendix)

We drew the Capital Allocation Line using the formula

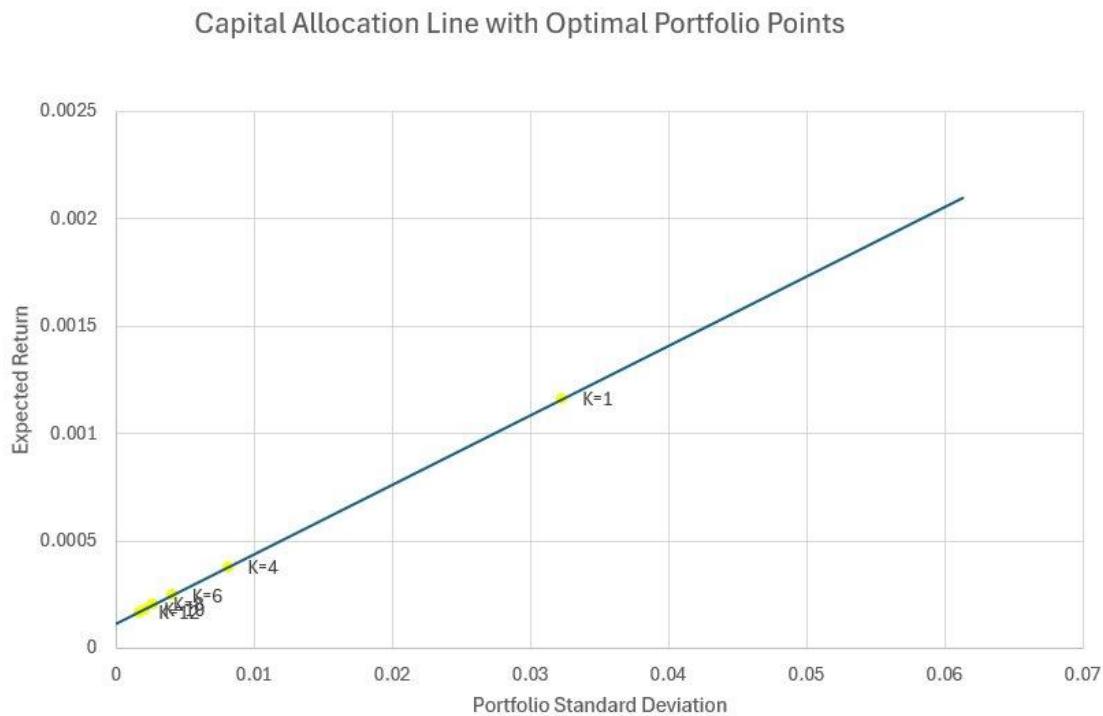
$E[r_c] = r_f + \text{Sharpe}_T \cdot \sigma_c$  where  $\sigma_c$  is the daily volatility of the chosen portfolio. It is the linear line that goes through our risk-free rate and intersects our tangency portfolio. Its slope is the Sharpe Ratio.



#### Appendix G: Tangency Portfolio and CAL

The solution used did not have any short selling, so our values are the same with or without short selling. Analysing these results, an investor's best combination of IDXX and MAR is to essentially be fully in MAR. There are benefits to diversification, for example, if we were to recommend a portfolio to someone who is very risk averse e.g. someone of retirement age, we would advise a stronger weighting of IDXX and a portfolio that looks more like the Minimum Variance Portfolio that we calculated in Q5.

## **Q7 The Allocation of a Portfolio between Risk-Free and Risky Assets.**



Appendix H: CAL with optimal portfolio points.

Where an investor's portfolio lies on the CAL depends on their attitude to risk. Older investors would likely be closer to the risk-free rate as they cannot afford to lose as much. Younger risk-seeking investors will choose a point further out on the CAL and may opt to borrow at the risk-free rate becoming more than 100% leveraged in risky assets. This stems from the utility function  $U = R_p - k/2 \times \sigma_p^2$  where  $k/2$  is a coefficient of risk aversion. The point where utility will be maximised is at the first differential of this function. When  $k$  is lower, the investor is risk-seeking and the optimal point will be further along the CAL with a higher

risk-return trade-off ( $k=1$ ). Here, the investor is borrowing at the risk-free rate and leveraging themselves beyond 100% in IDXX and MAR, as evidenced by column 4 in Appendix I where  $k=1$ . If an investor is risk-averse ( $k$  is larger) they will pick a point on CAL closer to the risk-free intercept as seen by the cluster of points closer to the origin on Appendix H. This observation aligns with classical portfolio theory studied by Kroll *et al* (1984). While the portfolio now includes the risk-free asset, the ratio between IDXX and MAR remains constant as the tangency portfolio composition does not change but the amount allocated to said tangency portfolio vs the risk-free asset depends on the investor's risk aversion.

k	sigmaT^2	y*	w_rf	w_IDXX	w_MAR	sigmap	r_p	Utility
1	0.0006011624325	1.3184223716858	-0.3184223716858	0.016109	1.302313	0.032325889	0.0011606451139	0.000638164
2	0.0006011624325	0.6592111858429	0.3407888141571	0.008054	0.651157	0.016162945	0.0006381635569	0.000376923
4	0.0006011624325	0.3296055929215	0.6703944070786	0.004027	0.325578	0.008081472	0.0003769227785	0.000246302
6	0.0006011624325	0.2197370619476	0.7802629380524	0.002685	0.217052	0.005387648	0.0002898425190	0.000202762
8	0.0006011624325	0.1648027964607	0.8351972035393	0.002014	0.162789	0.004040736	0.0002463023892	0.000180992
10	0.0006011624325	0.1318422371686	0.8681577628314	0.001611	0.130231	0.003232589	0.0002201783114	0.00016793
12	0.0006011624325	0.1098685309738	0.8901314690262	0.001342	0.108526	0.002693824	0.0002027622595	0.000159222
14	0.0006011624325	0.0941730265490	0.9058269734510	0.001151	0.093022	0.002308992	0.0001903222224	0.000153002
16	0.0006011624325	0.0824013982304	0.9175986017696	0.001007	0.081395	0.002020368	0.0001809921946	0.000148337
18	0.0006011624325	0.0732456873159	0.9267543126841	0.000895	0.072351	0.001795883	0.0001737355063	0.000144709
20	0.0006011624325	0.0659211185843	0.9340788814157	0.000805	0.065116	0.001616294	0.0001679301557	0.000141806

Appendix I: Utility Function Calculation

## Q8 CAPM Beta calculation using OLS Regression

Market Risk premium is calculated by finding the difference between the expected return on the overall market (S&P 500) and the risk-free interest rate. Beta is used to measure the stock's volatility or systematic risk in relation to the overall market. When beta is 1, the asset moves in tandem with the market. Beta greater than 1 means the asset is more volatile than the market,

and beta less than 1 means it's less volatile. A beta of zero implies that there is no correlation to market movements, such as government securities.

Formula to find CAPM Beta using OLS regression:

$$R_{\text{asset}} - R_f = \alpha + \beta (R_{\text{market}} - R_f) + \epsilon,$$

where our Y variable is the excess return of the asset, X variable is excess market return,  $\beta$  is slope coefficient measuring systematic risk,  $\alpha$  is intercept representing abnormal return,  $\epsilon$  is error term.

**OLS Regression:**

**IDEXX:**

**ANOVA Table:**

Source	DF	Sum of Squares	Mean Square	F - Value	P- value	R Square
Regression	1	0.20372	0.20372	750.57260	7.09526425957011E-130	0.374617122938942
Residual	1253	0.34008	0.00027			
Total	1254	0.54380				

**Coefficient Estimates:**

Term	Coefficients	Standard Error	T Stat	P-value	Lower 95%	Upper 95%
Intercept - Alpha	-0.00011	0.000465774	-0.23388033	0.81511609	-0.001022719	0.000804848
X Variable 1 Beta	1.16071	0.042367272	27.39658017	7.10E-130	1.077599754	1.243836987



**Appendix J: Regression analysis plot with beta**

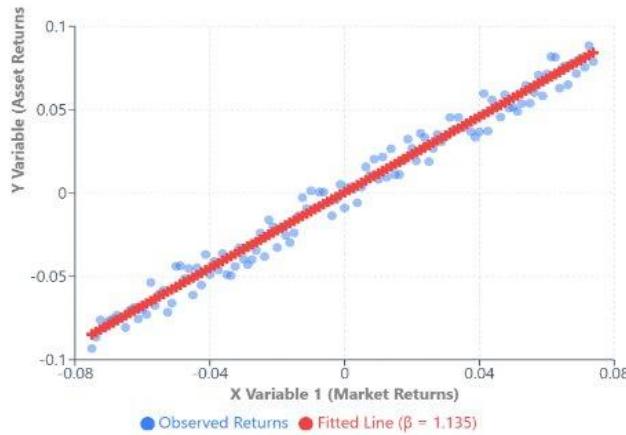
**MAR:**

**ANOVA Table:**

Source	DF	Sum of Squares	Mean Square	F - Value	P- value	R Square
Regression	1	0.19481	0.19481	835.08551	3.93927585097916E-141	0.399928792022455
Residual	1253	0.29231	0.00023			
Total	1254	0.48713				

**Coefficient Estimates:**

Term	Coefficients	Standard Error	T Stat	P-value	Lower 95%	Upper 95%
Intercept - Alpha	0.000421182	0.000431821	0.975364529	0.329567734	-0.000425989	0.001268353
X Variable 1 Beta	1.135073234	0.039278818	28.89784628	3.94E-141	1.05801373	1.212132738



**Appendix K: Regression analysis plot with beta for MAR**

### Interpretation:

- OLS Regression was used to derive the beta of both stocks. Beta of IDEXX is 1.16 or 16% and Beta of MAR is 1.13 or 13.5%, showing that IDEXX is more volatile.
- In comparison with  $R^2$ , IDEXX: 37.46% systematic risk and 62.54% idiosyncratic risk, and MAR: 39.99% systematic risk and 60.01% idiosyncratic risk. MAR is a better fit.
- Analysing F-value, T-stat, and CI width of both the stock, MAR has stronger relationship, more significant, and precise when compared to IDEXX.
- Both stocks are more volatile than market ( $\beta > 1$ )
- Highly significant market exposure which makes both the stocks suitable for growth-oriented portfolios
- Should be combined with lower-beta stocks for diversification or asset allocation if planning for long-term investment.

## Q9 CAPM Performance, Security Market Line Analysis

The Capital Asset Pricing Model (CAPM) was used to analyse the risk-return profile of IDXX and MAR, using daily data for July 2025. The first step involves estimating each stock's beta with OLS regression against market returns from the S&P 500, quantifying systematic risk. The predicted expected return for each stock is then calculated using the CAPM:

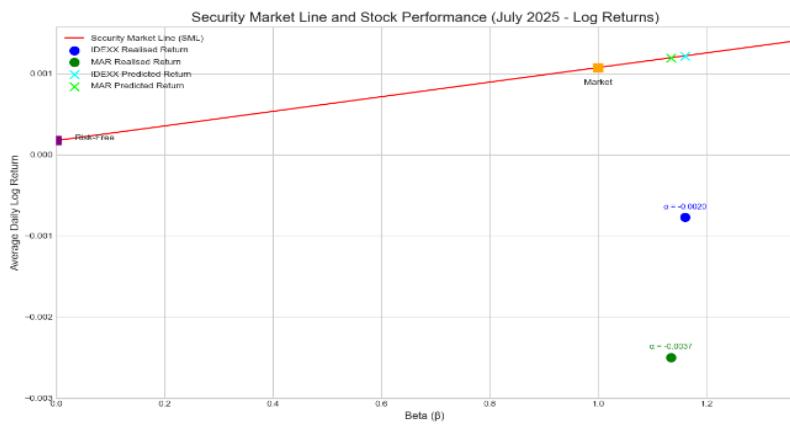
$$E(R_i) = R_f + \beta_i \cdot (E(R_m) - R_f)$$

Where: i)  $E(R_i)$  is the expected return of stock, ii)  $R_f$  is the average daily risk-free rate, iii)  $\beta_i$  the stock's beta, and  $E(R_m) - R_f$  is the average daily market risk premium.

<b>Market and Risk-Free Rate</b>	
<b>Average Daily Risk-Free Rate:</b>	0.000174
<b>Average Daily S&amp;P 500 (Market) Log Return:</b>	0.001074
<b>Average Daily Market Risk Premium:</b>	0.0009

**Jensen's Alpha ( $\alpha$ )**, a measure of abnormal performance that being the difference between the realized return and the CAPM-predicted return, assessing whether the stock outperformed/underperformed its benchmark. The results are visualized by plotting realized and predicted returns against beta with the Security Market Line (SML), show that IDXX and MAR underperformed; MAR experienced significant underperformance with a larger negative alpha, indicating compensation below its systematic risk exposure. The analysis highlights the need for further interpretation using CAPM and  $\alpha$  for evidence-based stock appraisal.

Stock Performance: IDEXX (IDXX)		Stock Performance: Marriott (MAR)	
Beta ( $\beta$ ):	1.1607	Beta ( $\beta$ ):	1.135
Average Daily Realised Log Return:	-0.000774	Average Daily Realised Log Return:	-0.002503
Predicted Log Return (CAPM):	0.001219	Predicted Log Return (CAPM):	0.001196
Jensen's Alpha ( $\alpha$ ):	-0.001993	Jensen's Alpha ( $\alpha$ ):	-0.003699



Appendix L: Security Market line with Actual and Expected returns plotted

## Q10 Bond Valuation, Duration Risk, Real-World Consequences

Question 10 focuses on fixed-income analysis. The scenario considers a \$1,000 investment in a 30-year U.S. Treasury bond purchased at par at a 1.90% yield (end of 2021). By end-2022, the yield doubled to 3.85%, prompting a substantial price decline. The bond's market value is re-estimated by discounting future semi-annual coupons and principal at the new yield, resulting in a value of \$661.13 and a net loss (including coupons) of \$319.87.

Duration analysis serves as a key risk estimator. Modified duration (here, 22.75 years) quantifies the price sensitivity to rate changes, with the percent price change calculated as:

$$\% \text{ Change} = -\text{Modified Duration} \times \Delta \text{Yield}$$

Applying this, the estimated price drop is -\$443.63—higher than the actual loss due to convexity, which moderates price declines as rates jump. The episode draws a parallel to Silicon Valley Bank's (SVB) collapse, where extensive duration risk and rate shocks led to catastrophic unrealized losses. The findings underscore that “safe” bonds carry latent risks in rising rate environments, stressing the relevance of duration, convexity, and yield risk for contemporary risk management.

## Conclusion

This report sought to show how investor preferences shape portfolio choices using quantitative methods. Analysis was conducted in a step-by-step method, calculating the returns and risks of both IDEXX and MAR, examining correlation, and showing how combining them produced diversification benefits. By deriving the minimum variance portfolio and graphing the efficient frontier, the weighting of IDEXX and MAR which provided the best risk and return trade-off was determined. The CAL was constructed including the risk-free asset which represented the highest possible Sharpe ratio attainable for investors. This report showed how while the CAL is the same for all investors, the final allocation is relative to the investor's attitude to risk. Our analysis found that risk-seeking investors may even leverage their position to invest more in the risky assets. While it is important to note that our analysis is limited in that this report only examined two risky assets, our analysis is in line with the traditional insights of portfolio theory such as how diversification reduces idiosyncratic risk, the efficient frontier defines a set of rational choices and why investors all share the same risky portfolio but hold it in different proportions. By applying these ideas to real companies such as IDEXX and MAR, our report has shown that theoretical models can provide a framework for making investment decisions when applied with empirical evidence and methodical analysis.

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## Appendix

### Q6 EXCEL WORKINGS

Description	Notes	Value	
Set $\mu_{IDXX}$		0.00037611	
Set $\mu_{MAR}$		0.00091485	
Set $\sigma_{IDXX}$		0.02078789	
Set $\sigma_{MAR}$		0.019645778	
Set Var_IDXX		0.00042136	
Set Var_MAR		0.000385957	
Set $r_f$ (daily)		0.000115682	
Correlation		0.299442603	
Covariance		0.000122193	
Build $\Sigma$	[[Var_IDXX, Cov], [Cov, Var_MAR]]	0.00042136	0.000122193
	This is a 2 x 2 matrix	0.000122193	0.000385957
Invert $\Sigma$		2613.190282	-827.329366
		-827.329366	2852.892569
Build $\mu$	$[\mu_{IDXX}, \mu_{MAR}]^T$	[0.00037611 0.00091485]	
$(\mu - r_f)$	Mean returns minus risk free rate.	0.000260428	0
	This is a 2x1 matrix	0.000799617	0
New Matrix Z	$\Sigma^{-1}(\mu - r_f)$	0.019001965	0
		2.065760986	0
Sum Z		2.084762951	
Tangency weights $w_T$	IDXX (Value in Matrix Z divided by Sum Z)	0.9114688%	IDXX
$z / (\sum z)$	MAR ((Value in Matrix Z divided by Sum Z)	99.0885312%	MAR
$\mu_T$	Mean return on tangency portfolio	0.00090994	
Variance on Tangency Portfolio		0.000378989	
Standard deviation on Tangency Portfolio		0.019467645	
Sharpe Ratio of Tangent Portfolio		0.04079886	

### PYTHON SCRIPT Q5 Q6

```
import numpy as np

import pandas as pd

import matplotlib.pyplot as plt
```

```

mu = np.array([0.00037611, 0.00091485])      # [IDXX, MAR]

sigma = np.array([0.02078789, 0.019645778]) # [IDXX, MAR]

rho = 0.299442597

rf = 0.000115682

TRADING_DAYS = 252

tickers = ["IDXX", "MAR"]

cov = np.array([
    [sigma[0]**2, rho * sigma[0] * sigma[1]],
    [rho * sigma[0] * sigma[1], sigma[1]**2],
])

Sigma_inv = np.linalg.inv(cov)

excess = mu - rf

w_unnorm = Sigma_inv @ excess

w_tan = w_unnorm / w_unnorm.sum()

mu_tan = float(w_tan @ mu)

sig_tan = float(np.sqrt(w_tan @ cov @ w_tan))

sharpe_daily = (mu_tan - rf) / sig_tan

w_idxx = np.linspace(-0.5, 1.5, 400)

w_mar = 1 - w_idxx

mus = w_idxx * mu[0] + w_mar * mu[1]

sigs = np.sqrt((w_idxx**2) * cov[0,0] + (w_mar**2) * cov[1,1] + 2 * w_idxx * w_mar * cov[0,1])

w_idxx_ns = np.linspace(0, 1, 200)

w_mar_ns = 1 - w_idxx_ns

```

```

mus_ns = w_idxx_ns * mu[0] + w_mar_ns * mu[1]

sigs_ns = np.sqrt(
    (w_idxx_ns**2) * cov[0,0] +
    (w_mar_ns**2) * cov[1,1] +
    2 * w_idxx_ns * w_mar_ns * cov[0,1]
)

sig_range = np.linspace(0, sig_tan * 1.6, 200)

mu_cal = rf + sharpe_daily * sig_range

summary = pd.DataFrame(
    {"Weight": w_tan, "Daily mean  $\mu$ ": mu, "Daily  $\sigma$ ": sigma},
    index=tickers
)

overview = pd.Series({
    "rf (daily)": rf,
    " $\mu_T$  (daily)": mu_tan,
    " $\sigma_T$  (daily)": sig_tan,
    "Sharpe (daily)": sharpe_daily,
    "rf (annual)": rf * TRADING_DAYS,
    "E[R]_T (annual)": mu_tan * TRADING_DAYS,
    " $\sigma_T$  (annual)": sig_tan * np.sqrt(TRADING_DAYS),
    "Sharpe (annual)": sharpe_daily * np.sqrt(TRADING_DAYS),
})

print("Tangency weights:\n", summary[["Weight"]])
print("\nOverview:\n", overview)

```

```

w1_mv = (sigma[1]**2 - cov[0,1]) / (sigma[0]**2 + sigma[1]**2 - 2*cov[0,1])

w_mv = np.array([w1_mv, 1 - w1_mv])      # [IDXX, MAR]

x_mv = np.sqrt(w_mv @ cov @ w_mv)      # stdev

y_mv = w_mv @ mu                      # mean

plt.figure(figsize=(8, 6))

plt.plot(sigs, mus, label="Efficient frontier")

plt.scatter([sigma[0]], [mu[0]], label="IDXX", marker="s")

plt.scatter([sigma[1]], [mu[1]], label="MAR", marker="s")

plt.scatter([sig_tan], [mu_tan], marker="x", s=100, label="Tangency portfolio")

plt.scatter([0], [rf], marker="s", label="Risk-free rate")

plt.plot(sig_range, mu_cal, linestyle=":", label="CAL")

plt.scatter([x_mv], [y_mv], marker="s", s=70, label="Minimum Variance Portfolio")

plt.xlabel("Daily volatility")

plt.ylabel("Daily expected return")

plt.title("Minimum Variance Portfolio")

plt.legend()

plt.grid(True)

plt.tight_layout()

plt.show()

```